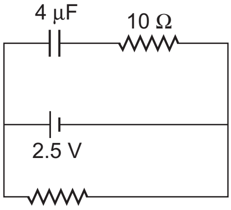
2

Electrostatic Potential and Capacitance

Multiple Choice Questions (MCQs)

Q. 1 A capacitor of 4 μF is connected as shown in the circuit. The internal resistance of the battery is 0.5Ω. The amount of charge on the capacitor plates will be



(a) 0 (b) 4 μ C (c) 16μC (d) 8 μC

K **Thinking Process**

In this problem, the three parallel branches of circuit can be considered in parallel , combination with one-another. Therefore, potential difference across each branch is same. The capacitor offers infinite resistance in DC circuit, therefore no current flows through capacitor and 10Ω resistance, leaving zero potential difference across 10Ω resistance.

Thus, potential difference across lower and middle branch of circuit is equal to the potential difference across capacitor of upper branch of circuit.

Ans. *(d)* Current flows through 2Ω resistance from left to right, is given by

$$I = \frac{V}{R + r} = \frac{2.5V}{2 + 0.5} = 1A$$

The potential difference across 2Ω resistance $V = IR = 1 \times 2 = 2V$

Since, capacitor is in parallel with 2Ω resistance, so it also has 2V potential difference across it.

The charge on capacitor

$$q = CV = (2\,\mu F) \times 2V = 8\mu C$$

Note *The potential difference across 2Ω resistance solely occurs across capacitor as no potential drop occurs across 10Ω resistance.*

Q. 2 A positively charged particle is released from rest in an uniform electric field. The electric potential energy of the charge

(a) remains a constant because the electric field is uniform
(b) increases because the charge moves along the electric field
(c) decreases because the charge moves along the electric field
(d) decreases because the charge moves opposite to the electric field

Thinking Process

In this problem, the relationship between E and V is actualised .

Ans. *(c)* The direction of electric field is always perpendicular to one equipotential surface maintained at high electrostatic potential to other equipotential surface maintained at low electrostatic potential.

The positively charged particle experiences electrostatic force along the direction of electric field *i.e.*, from high electrostatic potential to low electrostatic potential. Thus, the work is done by the electric field on the positive charge, hence electrostatic potential energy of the positive charge decreases.

Q. 3 Figure shows some equipotential lines distributed in space. A charged object is moved from point *A* to point *B*.

(a) The work done in Fig. (i) is the greatest
(b) The work done in Fig. (ii) is least
(c) The work done is the same in Fig. (i), Fig.(ii) and Fig. (iii)
(d) The work done in Fig. (iii) is greater than Fig. (ii) but equal to that in

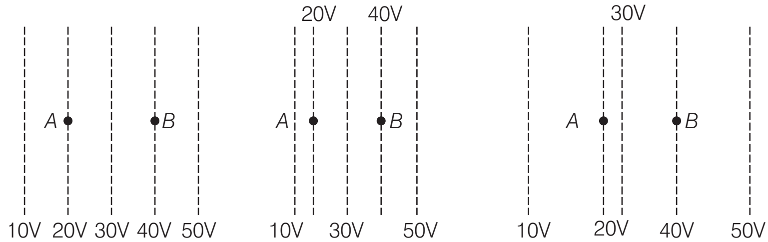


Fig. (i) **Fig. (ii)** **Fig. (iii)**

Ans. *(c)* The work done by a electrostatic force is given by $W_{12} = q(V_2 - V_1)$. Here initial and final potentials are same in all three cases and same charge is moved, so work done is same in all three cases.

Q. 4 The electrostatic potential on the surface of a charged conducting sphere is 100V. Two statements are made in this regard

S_1 at any point inside the sphere, electric intensity is zero.

S_2 at any point inside the sphere, the electrostatic potential is 100V.

Which of the following is a correct statement?

(a) S_1 is true but S_2 is false
(b) Both S_1 and S_2 are false
(c) S_1 is true, S_2 is also true and S_1 is the cause of S_2
(d) S_1 is true, S_2 is also true but the statements are independant

Ans. *(c)* In this problem, the electric field intensity E and electric potential V are related as

$$E = -\frac{dV}{dr}$$

Electric field intensity $E = 0$ suggest that $\frac{dV}{dr} = 0$

This imply that $V =$ constant.

Thus, $E = 0$ inside the charged conducting sphere causes , the same electrostatic potential 100V at any point inside the sphere.

Note *V equals zero does not necessary imply that $E = 0$ e.g., the electric potential at any point on the perpendicular bisector due to electric dipole is zero but E not.*

$E = 0$ does not necessary imply that $V = 0$ e.g., the electric field intensity at any point inside the charged spherical shell is zero but there may exist non-zero electric potential.

Q. 5 Equipotentials at a great distance from a collection of charges whose total sum is not zero are approximately

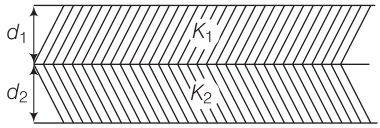
(a) spheres (b) planes
(c) paraboloids (d) ellipsoids

Ans. *(a)* In this problem, the collection of charges, whose total sum is not zero, with regard to great distance can be considered as a point charge. The equipotentials due to point charge are spherical in shape as electric potential due to point charge q is given by

$$V = k_e \frac{q}{r}$$

This suggest that electric potentials due to point charge is same for all equidistant points. The locus of these equidistant points, which are at same potential, form spherical surface.

Q. 6 A parallel plate capacitor is made of two dielectric blocks in series. One of the blocks has thickness d_1 and dielectric constant K_1 and the other has thickness d_2 and dielectric constant K_2 as shown in figure. This arrangement can be thought as a dielectric slab of thickness $d (= d_1 + d_2)$ and effective dielectric constant K. The K is`



(a) $\frac{K_1 d_1 + K_2 d_2}{d_1 + d_2}$ (b) $\frac{K_1 d_1 + K_2 d_2}{K_1 + K_2}$

(c) $\frac{K_1 K_2 (d_1 + d_2)}{(K_1 d_1 + K_2 d_2)}$ (d) $\frac{2K_1 K_2}{K_1 + K_2}$

K **Thinking Process**

In this problem, the system can be considered as the series combination of two capacitors which are of thicknesses d_1 and filled with dielectric medium of dielectric constant K_1 and thicknesses d_2 and filled with dielectric medium of dielectric constant K_2.

Ans. *(c)* The capacitance of parallel plate capacitor filled with dielectric block has thickness d_1 and dielectric constant K_2 is given by

$$C_1 = \frac{K_1 \varepsilon_0 A}{d_1}$$

Similarly, capacitance of parallel plate capacitor filled with dielectric block has thickness d_2 and dielectric constant K_2 is given by

$$C_2 = \frac{K_2 \varepsilon_0 A}{d_2}$$

Since, the two capacitors are in series combination, the equivalent capacitance is given by

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$

or

$$C = \frac{C_1 C_2}{C_1 + C_2} = \frac{\frac{K_1 \varepsilon_0 A}{d_1} \frac{K_2 \varepsilon_0 A}{d_2}}{\frac{K_1 \varepsilon_0 A}{d_1} + \frac{K_2 \varepsilon_0 A}{d_2}} = \frac{K_1 K_2 \varepsilon_0 A}{K_1 d_2 + K_2 d_1} \quad \text{...(i)}$$

But the equivalent capacitances is given by

$$C = \frac{K \varepsilon_0 A}{d_1 + d_2}$$

On comparing, we have

$$K = \frac{K_1 K_2 (d_1 + d_2)}{K_1 d_2 + K_2 d_1}$$

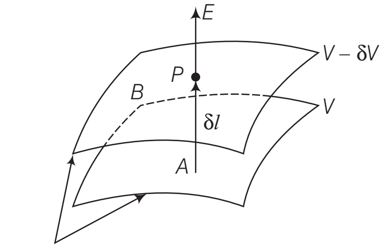
Note *For the equivalent capacitance of the combination, thickness is equal to the separation between two plates i.e., $d_1 + d_2$ and dielectric constant K.*

Multiple Choice Questions (More Than One Options)

Q. 7 Consider a uniform electric field in the $\hat{\mathbf{z}}$-direction. The potential is a constant

(a) in all space
(b) for any x for a given z
(c) for any y for a given z
(d) on the x-y plane for a given z

Ans. *(b, c, d)*



Here, the figure electric field is always remain in the direction in which the potential decreases steepest. Its magnitude is given by the change in the magnitude of potential per unit displacement normal to the equipotential surface at the point.

The electric field in z-direction suggest that equipotential surfaces are in x-y plane. Therefore the potential is a constant for any x for a given z, for any y for a given z and on the x-y plane for a given z.

Note *The shape of equipotential surfaces depends on the nature and type of distribution of charge e.g., point charge leads to produce spherical surfaces whereas line charge distribution produces cylindrical equipotential surfaces.*

Q. 8 Equipotential surfaces

(a) are closer in regions of large electric fields compared to regions of lower electric fields

(b) will be more crowded near sharp edges of a conductor

(c) will be more crowded near regions of large charge densities

(d) will always be equally spaced

K **Thinking Process**

In this problem, we need a relation between the electric field intensity E and electric potential V given by

$$E = -\frac{dV}{dr}$$

Ans. *(a, b, c)*

The electric field intensity E is inversely proportional to the separation between equipotential surfaces. So, equipotential surfaces are closer in regions of large electric fields.

Since, the electric field intensities is large near sharp edges of charged conductor and near regions of large charge densities. Therefore, equipotential surfaces are closer at such places.

Q. 9 The work done to move a charge along an equipotential from *A* to *B*

(a) cannot be defined as $-\int_A^B \mathbf{E}.\mathbf{dl}$

(b) must be defined as $-\int_A^B \mathbf{E}.\mathbf{dl}$

(c) is zero

(d) can have a non-zero value

Ans. *(c)* Work done in displacing a charge particle is given by $W_{12} = q(V_2 - V_1)$ and the line integral of electrical field from point 1 to 2 gives potential difference $V_2 - V_1 = -\int_1^2 E.dl$

For equipotential surface, $V_2 - V_1 = 0$ and $W = 0$.

Note *If displaced charged particle is + 1 C, then and only then option (b) is correct. But the NCERT exemplar book has given (b) as correct options which probably not so under given conditions.*

Q. 10 In a region of constant potential

(a) the electric field is uniform

(b) the electric field is zero

(c) there can be no charge inside the region

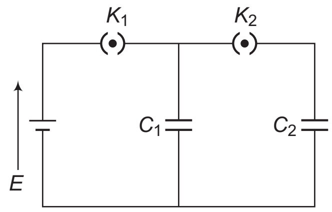
(d) the electric field shall necessarily change if a charge is placed outside the region

Ans. *(b, c)*

The electric field intensity E and electric potential V are related as $E = 0$ and for V = constant, $\frac{dV}{dr} = 0$

This imply that electric field intensity $E = 0$.

Q. 11 In the circuit shown in figure initially key K_1 is closed and key K_2 is open. Then K_1 is opened and K_2 is closed (order is important).
[Take Q_1' and Q_2' as charges on C_1 and C_2 and V_1 and V_2 as voltage respectively.]



Then,

(a) charge on C_1 gets redistributed such that $V_1 = V_2$
(b) charge on C_1 gets redistributed such that $Q_1' = Q_2'$
(c) charge on C_1 gets redistributed such that $C_1V_1 + C_2V_2 = C_1E$
(d) charge on C_1 gets redistributed such that $Q_1' + Q_2' = Q$

K **Thinking Process**

When key K_1 is closed and key K_2 is open, the capacitor C_1 is charged by cell and when K is opened and K_2 is closed, the charge stored by capacitor C_1 gets redistributed between C_1 and C_2.

Ans. *(a, d)*

The charge stored by capacitor C_1 gets redistributed between C_1 and C_2 till their potentials become same *i.e.*, $V_2 = V_1$. By law of conservation of charge, the charge stored in capacitor C_1 when key K_1 is closed and key K_2 is open is equal to sum of charges on capacitors C_1 and C_2 when K_1 is opened and K_2 is closed *i.e.*,

$$Q'_1 + Q'_2 = Q$$

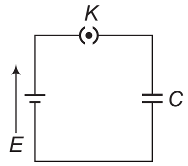
Q. 12 If a conductor has a potential $V \neq 0$ and there are no charges anywhere else outside, then

(a) there must be charges on the surface or inside itself
(b) there cannot be any charge in the body of the conductor
(c) there must be charges only on the surface
(d) there must be charges inside the surface

Ans. *(a, b)*

The charge resides on the outer surface of a closed charged conductor.

Q. 13 A parallel plate capacitor is connected to a battery as shown in figure. Consider two situations.



A. Key K is kept closed and plates of capacitors are moved apart using insulating handle.

B. Key K is opened and plates of capacitors are moved apart using insulating handle.

Choose the correct option(s).

(a) In **A** Q remains same but C changes

(b) In **B** V remains same but C changes

(c) In **A** V remains same and hence Q changes

(d) In **B** Q remains same and hence V changes

Thinking Process

The cell is responsible for maintaining potential difference equal to its emf across connected capacitor in every circumstance. However, charge stored by disconnected charged capacitor remains conserved.

Ans. ***(c , d)***

Case A When key K is kept closed and plates of capacitors are moved apart using insulating handle, the separation between two plates increases which in turn decreases its capacitance $\left(C = \frac{K\varepsilon_0 A}{d}\right)$ and hence, the charge stored decreases as $Q = CV$ (potential continue to be the same as capacitor is still connected with cell).

Case B When key K is opened and plates of capacitors are moved apart using insulating handle, charge stored by disconnected charged capacitor remains conserved and with the decreases of capacitance, potential difference V increases as $V = Q/C$.

Very Short Answer Type Questions

Q. 14 Consider two conducting spheres of radii R_1 and R_2 with $R_1 > R_2$. If the two are at the same potential, the larger sphere has more charge than the smaller sphere. State whether the charge density of the smaller sphere is more or less than that of the larger one.

Thinking Process

The electric potentials on spheres due to their charge need to be written in terms of their charge densities.

Ans. Since, the two spheres are at the same potential, therefore

$$\frac{kq_1}{R_1} = \frac{kq_2}{R_2} \Rightarrow \frac{kq_1 R_1}{4\pi R_1^2} = \frac{kq_2 R_2}{4\pi R_2^2}$$

or $$\sigma_1 R_1 = \sigma_2 R_2 \Rightarrow \frac{\sigma_1}{\sigma_2} = \frac{R_2}{R_1}$$

$$R_2 > R_1$$

This imply that $\sigma_1 > \sigma_2$.

The charge density of the smaller sphere is more than that of the larger one.

Q. 15 Do free electrons travel to region of higher potential or lower potential?

Ans. The free electrons experiences electrostatic force in a direction opposite to the direction of electric field being is of negative charge. The electric field always directed from higher potential to lower travel.

Therefore, electrostatic force and hence direction of travel of electrons is from lower potential to region of higher potential .

Q. 16 Can there be a potential difference between two adjacent conductors carrying the same charge?

K **Thinking Process**

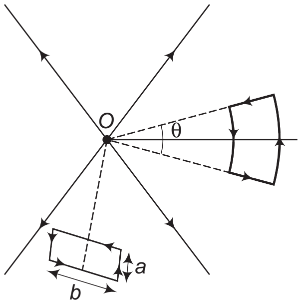
The capacity of conductor depend on its geometry i.e., length and breadth . For given charge potential $V \propto 1/C$, so two adjacent conductors carrying the same charge of different dimensions may have different potentials.

Ans. *Yes*, if the sizes are different.

Q.17 Can the potential function have a maximum or minimum in free space?

Ans. *No*, The absence of atmosphere around conductor prevents the phenomenon of electric discharge or potential leakage and hence, potential function do not have a maximum or minimum in free space.

Q. 18 A test charge q is made to move in the electric field of a point charge Q along two different closed paths [figure first path has sections along and perpendicular to lines of electric field. Second path is a rectangular loop of the same area as the first loop. How does the work done compare in the two cases?



Ans. As electric field is conservative, work done will be zero in both the cases.

Note *Conservative forces (like electrostatic force or gravitational force) are those forces, work done by which depends only on initial position and final position of object viz charge, but not on the path through which it goes from initial position to final position.*

Short Answer Type Questions

Q. 19 Prove that a closed equipotential surface with no charge within itself must enclose an equipotential volume.

K **Thinking Process**

In this problem, we need to know that the electric field intensity E and electric potential V are related as $E = -\frac{dV}{dr}$ and the field lines are always perpendicular to one equipotential surface maintained at high electrostatic potential to other equipotential surface maintained at low electrostatic potential.

Ans. Let's assume contradicting statement that the potential is not same inside the closed equipotential surface. Let the potential just inside the surface is different to that of the surface causing in a potential gradient $\left(\frac{dV}{dr}\right)$. Consequently electric field comes into existence, which is given by as $E = -\frac{dV}{dr}$.

Consequently field lines pointing inwards or outwards from the surface. These lines cannot be again on the surface, as the surface is equipotential. It is possible only when the other end of the field lines are originated from the charges inside.

This contradict the original assumption. Hence, the entire volume inside must be equipotential.

Q. 20 A capacitor has some dielectric between its plates and the capacitor is connected to a DC source. The battery is now disconnected and then the dielectric is removed . State whether the capacitance, the energy stored in it, electric field, charge stored and the voltage will increase, decrease or remain constant.

K **Thinking Process**

Here, the charge stored by the capacitor remains conserved after its disconnection from battery.

Ans. The capacitance of the parallel plate capacitor, filled with dielectric medium of dielectric constant K is given by

$$C = \frac{K\varepsilon_0 A}{d}, \text{ where signs are as usual.}$$

The capacitance of the parallel plate capacitor decreases with the removal of dielectric medium as for air or vacuum $K = 1$.

After disconnection from battery charge stored will remain the same due to conservation of charge.

The energy stored in an isolated charge capacitor $= \frac{q^2}{2C}$; as q is constant, energy stored $\propto$ $1/C$ and C decreases with the removal of dielectric medium, therefore energy stored increases. Since q is constant and $V = q/C$ and C decreases which in turn increases V and therefore E increases as $E = V/d$.

Note *One of the very important questions with the competitive point of view.*

Q. 21 Prove that, if an insulated, uncharged conductor is placed near a charged conductor and no other conductors are present, the uncharged body must intermediate in potential between that of the charged body and that of infinity.

Thinking Process

The electric field $E = -\frac{dV}{dr}$ suggest that electric potential decreases along the direction of electric field.

Ans. Let us take any path from the charged conductor to the uncharged conductor along the direction of electric field. Therefore, the electric potential decrease along this path.

Now, another path from the uncharged conductor to infinity will again continually lower the potential further. This ensures that the uncharged body must be intermediate in potential between that of the charged body and that of infinity.

Q. 22 Calculate potential energy of a point charge $-q$ placed along the axis due to a charge $+Q$ uniformly distributed along a ring of radius R. Sketch PE, as a function of axial distance z from the centre of the ring. Looking at graph, can you see what would happen if $-q$ is displaced slightly from the centre of the ring (along the axis)?

Thinking Process

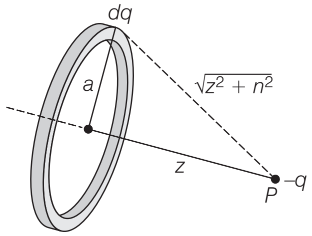
The work done or PE stored in a system of charges can be obtained
$U = W = q \times$ potential difference

Ans. Let us take point P to be at a distance x from the centre of the ring, as shown in figure. The charge element dq is at a distance x from point P. Therefore, V can be written as

$$V = k_e \int \frac{dq}{r} = k_e \int \frac{dq}{\sqrt{z^2 + a^2}}$$

where, $k = \frac{1}{4\pi\varepsilon_0}$, since each element dq is at the same distance from point P, so we have net potential

$$V = \frac{k_e}{\sqrt{z^2 + a^2}} \int dq = \frac{k_e Q}{\sqrt{z^2 + a^2}}$$

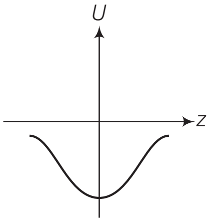


Considering $-q$ charge at P, the potential energy is given by

$$U = W = q \times \text{potential difference}$$

$$U = \frac{k_e Q(-q)}{\sqrt{z^2 + a^2}}$$

or
$$U = \frac{1}{4\pi\varepsilon_0} \frac{-Qq}{\sqrt{z^2 + a^2}}$$
$$= \frac{1}{4\pi\varepsilon_0 a} \frac{-Qq}{\sqrt{1 + \left(\frac{z}{a}\right)^2}}$$



This is the required expression.

The variation of potential energy with z is shown in the figure. The charge $-q$ displaced would perform oscillations.

Nothing can be concluded just by looking at the graph.

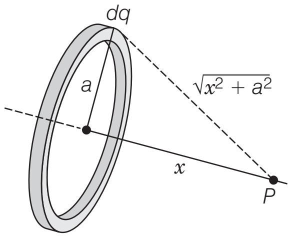
Q. 23 Calculate potential on the axis of a ring due to charge Q uniformly distributed along the ring of radius R.

Ans. Let us take point P to be at a distance x from the centre of the ring,as shown in figure. The charge element dq is at a distance x from point P. Therefore, V can be written as

$$V = k_e \int \frac{dq}{r} = k_e \int \frac{dq}{\sqrt{x^2 + a^2}}$$

where, $k_e = \frac{1}{4\pi\varepsilon_0}$, since each element dq is at the same distance from point P, so we have net potential

$$V = \frac{k_e}{\sqrt{x^2 + a^2}} \int dq = \frac{k_e Q}{\sqrt{x^2 + a^2}}$$



The net electric potential $V = \frac{1}{4\pi\varepsilon_0} \frac{Q}{\sqrt{x^2 + a^2}}$

Long Answer Type Questions

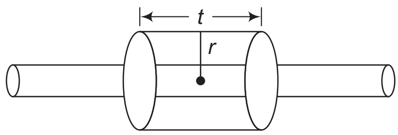
Q. 24 Find the equation of the equipotentials for an infinite cylinder of radius r_0 carrying charge of linear density λ.

Thinking Process

The electric field due to line charge need to be obtained in order to find the potential at distance r from the line charge. As line integral of electric field gives potential difference between two points.

$$V(r) - V(r_0) = -\int_{r}^{r} E.dl$$

Ans. Let the field lines must be radically outward. Draw a cylindrical Gaussian surface of radius r and length l. Then, applying Gauss' theorem



$$\int E.dS = \frac{1}{\varepsilon_0}\lambda l$$

or
$$E_r 2\pi r l = \frac{1}{\varepsilon_0}\lambda l \Rightarrow E_r = \frac{\lambda}{2\pi\varepsilon_0 r}$$

Hence, if r_0 is the radius,
$$V(r) - V(r_0) = -\int_{r_0}^{r} E.dl = \frac{\lambda}{2\pi\varepsilon_0}\ln\frac{r_0}{r}$$

Since,
$$\int_{r_0}^{r}\frac{\lambda}{2\pi\varepsilon_0 r}dr = \frac{\lambda}{2\pi\varepsilon_0}\int_{r_0}^{r}\frac{1}{r}dr = \frac{\lambda}{2\pi\varepsilon_0}\ln\frac{r}{r_0}$$

For a given V,

$$\ln\frac{r}{r_0} = -\frac{2\pi\varepsilon_0}{\lambda}[V(r) - V(r_0)]$$

$\Rightarrow$
$$r = r_0 e^{-2\pi\varepsilon_0 V r_0/\lambda} e + 2\pi\varepsilon_0 V(r)/\lambda$$

$$r = r_0 e^{-2\pi\varepsilon_0 [V(r) - V(r_0)]/\lambda}$$

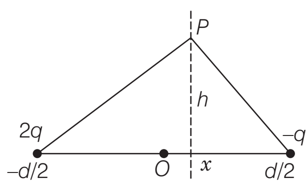
The equipotential surfaces are cylinders of radius.

Q. 25 Two point charges of magnitude $+q$ and $-q$ are placed at $(-d/2, 0, 0)$ and $(d/2, 2, 0)$, respectively. Find the equation of the equipotential surface where the potential is zero.

Thinking Process

The net electric potential at any point due to system of point charges is equal to the algebraic sum of electric potential due to each individual charges.

Ans. Let the required plane lies at a distance x from the origin as shown in figure.



The potential at the point P due to charges is given by

$$\frac{1}{4\pi\varepsilon_0}\frac{q}{[(x+d/2)^2+h^2]^{1/2}}-\frac{1}{4\pi\varepsilon_0}\frac{q}{[(x-d/2)^2+h^2]^{1/2}}$$

If net electric potential is zero, then

$$\frac{1}{[(x+d/2)^2+h^2]^{1/2}}=\frac{1}{[(x-d/2)+h^2]^{1/2}}$$

Or $$(x-d/2)^2+h^2=(x+d/2)^2+h^2$$

$\Rightarrow$ $$x^2-dx+d^2/4=x^2+dx+d^2/4$$

Or $$2dx=0 \Rightarrow x=0$$

The equation of the required plane is $x=0$ *i.e.*, y-z plane.

Q. 26 A parallel plate capacitor is filled by a dielectric whose relative permittivity varies with the applied voltage (U) as $\varepsilon=\alpha U$ where $a=2V^{-1}$. A similar capacitor with no dielectric is charged to $U_0=78$ V. It is then connected to the uncharged capacitor with the dielectric. Find the final voltage on the capacitors.

K **Thinking Process**

In this problem, the dielectric of variable permittivity is used which gives new insight in the ordinary problem.

Ans. Assuming the required final voltage be U. If C is the capacitance of the capacitor without the dielectric, then the charge on the capacitor is given by $Q_1=CU$

Since, the capacitor with the dielectric has a capacitance εC. Hence, the charge on the capacitor is given by

$$Q_2=\varepsilon CU=(\alpha U)CU=\alpha CU$$

The initial charge on the capacitor is given by

$$Q_0=CU_0$$

From the conservation of charges, $$Q_0=Q_1+Q_2$$

Or $$CU_0=CU+\alpha CU^2$$

$\Rightarrow$ $$\alpha U^2+U-U_0=0$$

$\therefore$ $$U=\frac{-1\pm\sqrt{1+4\alpha U_0}}{2\alpha}$$

On solving for $U_0=78\,V$ and $a=2/V$, we get

$$U=6\text{V}$$

Q. 27 A capacitor is made of two circular plates of radius R each, separated by a distance $d<<R$. The capacitor is connected to a constant voltage. A thin conducting disc of radius $r<<R$ and thickness $t<<r$ is placed at a centre of the bottom plate. Find the minimum voltage required to lift the disc if the mass of the disc is m.

K **Thinking Process**

The disc will be lifted when weight is balanced by electrostatic force.

Ans. Assuming initially the disc is in touch with the bottom plate, so the entire plate is a equipotential.

The electric field on the disc, when potential difference V is applied across it, given by

$$E = \frac{V}{d}$$

Let charge q' is transferred to the disc during the process,

Therefore by Gauss' theorem,

$$\therefore \quad q' = -\varepsilon_0 \frac{V}{d} \pi r^2$$

Since, Gauss theorem states that $\phi = \frac{q}{\varepsilon_0}$ or $q = \frac{\varepsilon_0}{\phi}$

$$= \varepsilon EA = \frac{\varepsilon_0 V}{d} A$$

The force acting on the disc is

$$-\frac{V}{d} \times q' = \varepsilon_0 \frac{V^2}{d^2} \pi r^2$$

If the disc is to be lifted, then

$$\varepsilon_0 \frac{V^2}{d^2} \pi r^2 = mg \quad \Rightarrow \quad V = \sqrt{\frac{mgd^2}{\pi \varepsilon_0 r^2}}$$

This is the required expression.

Q. 28 (a) In a quark model of elementary particles, a neutron is made of one up quarks [charge (2/3) e] and two down quarks [charges $-(1/3)\,e$]. Assume that they have a triangle configuration with side length of the order of 10^{-15} m. Calculate electrostatic potential energy of neutron and compare it with its mass 939 MeV.

(b) Repeat above exercise for a proton which is made of two up and one down quark.

Ans. This system is made up of three charges. The potential energy of the system is equal to the algebraic sum of PE of each pair. So,

$$U = \frac{1}{4\pi\varepsilon_0}\left\{\frac{q_d q_d}{r} - \frac{q_u q_d}{r} - \frac{q_u q_d}{r}\right\}$$

$$= \frac{9 \times 10^9}{10^{-15}} (1.6 \times 10^{-19})^2 \; [\{(1/3)^2 - (2/3)(1/3) - (2/3)(1/3)\}]$$

$$= 2.304 \times 0^{-13} \left\{\frac{1}{9} - \frac{4}{9}\right\} = -7.68 \times 10^{-14} \text{J}$$

$$= 4.8 \times 10^5 \text{ eV} = 0.48 \text{ meV} = 5.11 \times 10^{-4} (m_n c^2)$$

Q. 29 Two metal spheres, one of radius R and the other of radius $2R$, both have same surface charge density σ. They are brought in contact and separated. What will be new surface charge densities on them?

Ans. The charges on two metal spheres, before coming in contact, are given by

$$Q = \sigma . 4\pi R^2$$

$$Q_2 = \sigma . 4\pi (2R^2)$$

$$= 4(\sigma . 4\pi R^2) = 4Q_1$$

Let the charges on two metal spheres , after coming in contact becomes Q_1' and Q_2'.

Now applying law of conservation of charges is given by

$$Q_1' + Q_2' = Q_1 + Q_2 = 5Q_1$$
$$= 5(\sigma.4\pi R^2)$$

After coming in contact, they acquire equal potentials. Therefore, we have

$$\frac{1}{4\pi\varepsilon_0}\frac{Q_1'}{R} = \frac{1}{4\pi\varepsilon_0}\frac{Q_2'}{R}$$

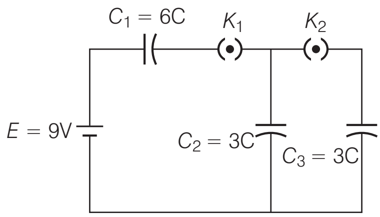
On solving, we get

$\therefore \quad Q_1' = \frac{5}{3}(\sigma.4\pi R^2)$ and $Q_2' = \frac{10}{3}(\sigma.4\pi R^2)$

$\therefore \quad \sigma_1 = 5/3\sigma$ and

$\therefore \quad \sigma_2 = \frac{5}{6}\sigma$

Q. 30 In the circuit shown in figure, initially K_1 is closed and K_2 is open. What are the charges on each capacitors?
Then K_1 was opened and K_2 was closed (order is important), what will be the charge on each capacitor now? $[C = 1\mu F]$



Ans. In the circuit, when initially K_1 is closed and K_2 is open, the capacitors C_1 and C_2 acquires potential difference V_1 and V_2 respectively. So, we have

$$V_1 + V_2 = E$$

and $\quad V_1 + V_2 = 9V$

Also, in series combination , $\quad V \propto 1/C$

$$V_1 : V_2 = 1/6 : 1/3$$

On solving

$\Rightarrow \quad V_1 = 3V$ and $V_2 = 6V$

$\therefore \quad Q_1 = C_1V_1 = 6C \times 3 = 18\,\mu C$

$$Q_2 = 9\,\mu C \text{ and } Q_3 = 0$$

Then, K_1 was opened and K_2 was closed, the parallel combination of C_2 and C_3 is in series with C_1.

$$Q_2 = Q_2' + Q_3$$

and considering common potential of parallel combination as V, then we have

$$C_2V + C_3V = Q_2$$

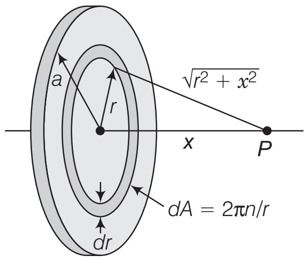
$\Rightarrow \quad V = \frac{Q_2}{C_2 + C_3} = (3/2)V$

On solving, $\quad Q_2' = (9/2)\,\mu C$

and $\quad Q_3 = (9/2)\,\mu C$

Q. 31 Calculate potential on the axis of a disc of radius R due to a charge Q uniformly distributed on its surface.

Ans. Let the point P lies at a distance x from the centre of the disk and take the plane of the disk to be perpendicular to the $\mathbf{x}$-axis. Let the disc is divided into a number of charged rings as shown in figure.



The electric potential of each ring, of radius r and width dr, have charge dq is given by

$$\sigma\, dA = \sigma 2\pi r dr$$

and potential is given by

(*Refer the solution of Q.* 23)

$$dV = \frac{k_e dq}{\sqrt{r^2 + x^2}} = \frac{k_e \sigma 2\pi r\, dr}{\sqrt{r^2 + x^2}}$$

where $k_e = \frac{1}{4\pi\varepsilon_0}$ the total electric potential at P, is given by

$$V = \pi k_e \sigma \int_0^a \frac{2r\, dr}{\sqrt{r^2 + x^2}} = \pi k_e \sigma \int_0^a (r^2 + x^2)^{-1/2} 2r\, dr$$

$$V = 2\pi k_e \sigma\, [(x^2 + a^2)^{1/2} - x]$$

So, we have by substring $\quad k_e = \frac{1}{4\pi\varepsilon_0}$

$$V = \frac{1}{4\pi\varepsilon_0} \frac{2Q}{a^2} [\sqrt{x^2 + a^2} - x]$$

Note *You may take $a = R$ in this problem.*

Q. 32 Two charges q_1 and q_2 are placed at $(0, 0, d)$ and $(0, 0, -d)$ respectively. Find locus of points where the potential is zero.

Thinking Process

Here, 3-dimensional imagination is required to actualise the problem. Also, the net electric potential at any point due to system of point charges is equal to the algebraic sum of electric potential due to each individual charges.

Ans. Let any arbitrary point on the required plane is (x, y, z) .The two charges lies on z-axis at a separation of $2d$.

The potential at the point P due to two charges is given by

$$\frac{q_1}{\sqrt{x^2 + y^2 + (z-d)^2}} + \frac{q_2}{\sqrt{x^2 + y^2 + (z+d)^2}} = 0$$

$$\therefore \quad \frac{q_1}{\sqrt{x^2 + y^2 + (z-d)^2}} = \frac{-q_2}{\sqrt{x^2 + y^2 + (z+d)^2}}$$

On squaring and simplifying, we get

$$x^2 + y^2 + z^2 + \left[\frac{(q_1/q_2)^2 + 1}{(q_1/q_2)^2 - 1}\right](2zd) + d^2 - 0$$

This is the equation of a sphere with centre at

$$\left(0, 0, -2d\left[\frac{q_1^2 + q_2^2}{q_1^2 - q_2^2}\right]\right)$$

Note *The centre and radius of sphere* $(x-a)^2 + (y-b)^2 + (z-c)^2 = r^2$ *is* (a, b, c) *and* r *respectively.*

Q. 33 Two charges $-q$ each are separated by distance $2d$. A third charge $+q$ is kept at mid-point O. Find potential energy of $+q$ as a function of small distance x from O due to $-q$ charges. Sketch PE Vs/x and convince yourself that the charge at O is in an unstable equilibrium.

Ans. Let third charge $+q$ is slightly displaced from mean position towards first charge. So, the total potential energy of the system is given by

$$U = \frac{1}{4\pi\varepsilon_0}\left\{\frac{-q^2}{(d-x)} + \frac{-q^2}{(d+x)}\right\}$$

$$U = \frac{-q^2}{4\pi\varepsilon_0}\frac{2d}{(d^2 - x^2)}$$

$$\frac{dU}{dx} = \frac{-q^2 2d}{4\pi\varepsilon_0}\cdot\frac{2x}{(d^2 - x^2)^2}$$

The system will be in equilibrium, if

$$F = -\frac{dU}{dx} = 0$$

On solving, $x = 0$. So for, $+q$ charge to be in stable/unstable equilibrium, finding second derivative of PE.

$$\frac{d^2U}{dx^2} = \left(\frac{-2dq^2}{4\pi\varepsilon_0}\right)\left[\frac{2}{(d^2 - x^2)^2} - \frac{8x^2}{(d^2 - x^2)^3}\right]$$

$$= \left(\frac{-2dq^2}{4\pi\varepsilon_0}\right)\frac{1}{(d^2 - x^2)^3}[2(d^2 - x^2)^2 - 8x^2]$$

At $x = 0$

$$\frac{d^2U}{dx^2} = \left(\frac{-2dq^2}{4\pi\varepsilon_0}\right)\left(\frac{1}{d^6}\right)(2d^2), \text{ which is } < 0$$

This shows that system will be unstable equilibrium.

Note *For function* $y = f(x)$, *on solving* $\frac{dy}{dx} = 0$ *gives critical points i.e., points of local maxima or local minima. If for any critical point, this imply that y acquires maximum value at* $x = x_1$, $x = x_1$

$\frac{d^2y}{dx^2} > 0$ *this imply that y acquires minimum value at* $x = x_1$ *and for* $\frac{d^2y}{dx^2} < 0$